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Thus, $2\cos\left(\frac{\alpha+\beta}{2}\right) - \cos\left(\frac{\alpha-\beta}{2}\right) = 0$ —— (1)

and $\sin\left(\frac{\alpha-\beta}{2}\right) = 0 \Rightarrow \frac{\alpha-\beta}{2} = 0$ $\left(\begin{array}{c} 0<\alpha,\beta<\pi \\ \text{Thus, can't be } \pi, 2\pi \end{array}\right)$

$\Rightarrow \alpha = \beta$

putting in eq (1)

$2\cos\alpha - 1 = 0$

$\cos\alpha = \frac{1}{2} \Rightarrow \alpha = \beta = 60^\circ \quad (0<\alpha,\beta<\pi)$

(Using, $\cos a + \cos b = 2\cos(a+b/2)\cos(a-b/2)$,
$a^2+b^2-2ab = (a-b)^2$,
if $A^2+B^2=0 \Rightarrow$ if and only if, $A=B=0$)

Q. $0<\beta<\frac{\pi}{2}$; $\sin\beta+\cos\beta+\tan\beta+\cot\beta+\sec\beta+\operatorname{cosec}\beta = 7$

then $\sin 2\beta$ is root of eq : a) $x^2-44x+36=0$
b) $x^2-44x+3=0$
c) $x^2-74x+3=0$
d) None

$\Rightarrow \sin\beta+\cos\beta+\frac{\sin\beta}{\cos\beta}+\frac{\cos\beta}{\sin\beta}+\frac{1}{\cos\beta}+\frac{1}{\sin\beta}=7$

$\Rightarrow \sqrt{1+\sin 2\beta} + \frac{1}{\sin\beta\cos\beta} + \frac{\sin\beta+\cos\beta}{\sin\beta\cos\beta} = 7$ $\left(|\sin\theta+\cos\theta| = \sqrt{1+\sin 2\theta}\right)$

$\sqrt{1+\sin 2\beta} + \frac{2}{\sin 2\beta} + \frac{2\sqrt{1+\sin 2\beta}}{\sin 2\beta} = 7$ $\left(\sin\beta\cos\beta = \frac{\sin 2\beta}{2}\right)$

Let $\sin 2\beta = x$ $\left(x \neq 0 \ (0<\beta<\pi/2)\right)$

simplifying

$\sqrt{1+x} + \frac{2}{x} + \frac{(\sqrt{1+x})\cdot 2}{x} = 7$

$x(x^2-44x+36)=0$

$\Rightarrow x=0$ (invalid) OR $x^2-44x+36=0$ (True) (a)

$$\Rightarrow \quad 3\sin\theta - 4\sin^3\theta = 4\sin\theta\,(\sin 2\theta \,\sin 4\theta)$$

$\sin\theta = 0$

$\theta = n\pi$

$\theta = \{0, \pi\}$

$$\Rightarrow 3 - 4\sin^2\theta = 2\,(2\sin 2\theta \sin 4\theta)$$

$$3 - 4\left(\frac{1-\cos 2\theta}{2}\right) = 2\,(\cos(2\theta) - \cos 6\theta)$$

$$= 3 - 2 + 2\cos 2\theta = 2\cos 2\theta - 2\cos 6\theta$$

$$\cos 6\theta = -\frac{1}{2} = \cos\left(\frac{2\pi}{3}\right)$$

$\theta \in [0, \pi]$

$$\theta = \frac{n\pi}{3} \pm \frac{\pi}{9} \quad (n \in Z)$$

$$\therefore 0 \le \frac{n\pi}{3} \pm \frac{\pi}{9} \le \pi$$

$$0 \le \frac{n\pi}{3} + \frac{\pi}{9} \le \pi \qquad\qquad 0 \le \frac{n\pi}{3} - \frac{\pi}{9} \le \pi$$

$$-\frac{1}{3} \le n \le \frac{8}{3} \qquad\qquad \frac{1}{3} \le n \le \frac{10}{3}$$

(integral values) $\rightarrow$ $n \in \{0,1,2\}$ $\qquad$ $n \in \{1,2,3\}$

Total no. of sol^n $= 2+3+3 = 8$

(4) Boundness

Q. $\sin x\left(\cos\left(\frac{x}{4}\right) - 2\sin x\right) + \left(1 + \sin\frac{x}{4} - 2\cos x\right)\cdot \cos x = 0$

$$\Rightarrow \sin x \cos\frac{x}{4} + \cos x \sin\frac{x}{4} - 2\sin^2 x - 2\cos^2 x + \cos x = 0$$

$$\sin\left(x + \frac{x}{4}\right) + \cos x = 2$$

$$\le 1 \quad + \quad \le 1 \rightarrow \le 2$$

Thus, $\sin\frac{5x}{4} = 1$ and $\cos x = 1$

$$\frac{5x}{4} = 2n\pi + \frac{\pi}{2} \quad \cap \quad x = 2n\pi$$

$$x = \frac{2\pi}{5}(4n+1)$$

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Q The no. of sol^n of equation, $[x] \equiv$ GIF

Domain Analysis

$\sin^{-1}[x^2+\frac{1}{3}] + \cos^{-1}[x^2-\frac{2}{3}] = x^2$ for $x \in [-1,1]$

$\Rightarrow$ $-1 \le [x^2+\frac{1}{3}] \le 1$ AND $-1 \le [x^2-\frac{2}{3}] \le 1$

$\underbrace{-1 \le x^2+\frac{1}{3}}_{\text{Always true}} < 2$ AND $\underbrace{-1 \le x^2-\frac{2}{3}}_{\text{Always true}} < 2$

$x^2 < 2-\frac{1}{3}$ & $x^2 < 2+\frac{2}{3}$

$\underbrace{0 \le x^2 < \frac{5}{3} \quad \& \quad 0 \le x^2 < \frac{8}{3}}$

$0 \le x^2 < \frac{5}{3}$

Case I:

$0 \le x^2 < \frac{2}{3}$

$\frac{1}{3} \le x^2+\frac{1}{3} < 1$ | $-\frac{2}{3} \le x^2-\frac{2}{3} < 0$

$[x^2+\frac{1}{3}]=0$ | $[x^2-\frac{2}{3}]=-1$

LHS $= \sin^{-1}0 + \cos^{-1}(-1) = 0+\pi = \pi$

$x^2 = \pi$ (rejected)

Case II:

$2/3 \le x^2 < 5/3$

$1 \le x^2+\frac{1}{3} < 2$ | $0 \le x^2-\frac{2}{3} < 1$

$[x^2+\frac{1}{3}]=1$ | $[x^2-\frac{2}{3}]=0$

LHS $= \sin^{-1}(1) + \cos^{-1}(0)$

$= \pi/2 + \pi/2 = \pi = x^2$

No solution

(rejected)

Q If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$ where $-1 \le x \le 1$, $-2 \le y \le 2$, $x \le \frac{y}{2}$ then $\forall x,y$, $4x^2 - 4xy\cos\alpha + y^2 = ?$

$\Rightarrow$

$\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$

M-III

M-I $\rightarrow$ use $\cos^{-1}x \pm \cos^{-1}y$ formula

M-II $\rightarrow$ convert ~~int~~ tan then use $\tan^{-1}x \pm \tan^{-1}y$ formula

$\Rightarrow A - B = \alpha$

$\cos A = x \rightarrow \sin A = \sqrt{1-x^2}$

$\cos B = \frac{y}{2} \rightarrow \sin B = \sqrt{1-\left(\frac{y}{2}\right)^2}$

$\cos(A-B) = \cos\alpha$

$\cos A\cos B + \sin A\sin B = \cos\alpha$

$\frac{xy}{2} + \sqrt{1-x^2}\sqrt{1-\frac{y^2}{4}} = \cos\alpha$

Q. If angle A, B and C of a Δ are in A.P and if a, b, c denote length opp to A, B, C resp, then value of $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A = ?$

$\Rightarrow \frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$

$A+B+C = \pi \quad (\Delta)$

$2B = A + C \quad (A, B, C \text{ in A.P})$

$\Downarrow$

$\Rightarrow B = \frac{\pi}{3}; \; A+C = \frac{2\pi}{3}$

from sine rule

$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{a}{c} = \frac{\sin A}{\sin C}$

$= \frac{\sin A}{\cancel{\sin C}} \times \cancel{2\sin C\cos C} + \frac{\sin C}{\sin A} \times 2\sin A\cos A$

$\Rightarrow 2(\sin A\cos C + \sin C\cos A)$

$\Rightarrow 2(\sin(A+C))$

$\Rightarrow 2(\sin(2\pi/3))$

$\Rightarrow \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}}$

$\Rightarrow \sqrt{3}$ Ans

Q. A circle touches the side BC of ΔABC at P and sides AB and AC produced at Q and R resp, If $\Sigma(b+c)\cos A = 5$ compute length (AQ)?

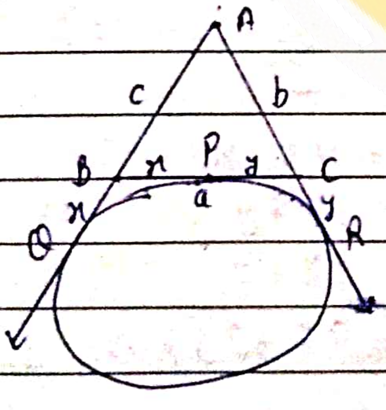
$\Rightarrow \Sigma(b+c)\cos A = 5$

$\Rightarrow (b+c)\cos A + (a+c)(\cos B) + (a+b)\cos C = 5$

$\Rightarrow b\cos A + c\cos A + a\cos B + c\cos B + a\cos C + b\cos C = 5$

$\Rightarrow \underline{b\cos A + a\cos B} + \underline{c\cos A + a\cos C} + \underline{b\cos C + c\cos B} = 5$

$\Rightarrow c + b + a = 5$



Through Geometry,

$BP = BQ = x, \; CP = CR = y, \; AQ = AR, \; BC = x+y$

$AQ = x + c \quad -(1)$

$AR = b + y \quad -(2)$

Adding, $2AQ = b + c + (x+y)$

$= b + c + a = 5$

$\therefore AQ = 5/2$ units

Q. Consider ΔABC, a, b, c denote length of side opp to A, B, C resp, Given that, $a = 6$, $b = 10$ and area of $\Delta = 15\sqrt{3}$, $\angle ACB$ is obtuse, if r denotes radius of incircle of Δ then $r^2 = ?$

Q Let total no of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is?

$$\Rightarrow \begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 2x & 4x^2 & 8x^3 \\ 3x & 9x^2 & 27x^3 \end{vmatrix}$$

$C_1 \leftrightarrow C_3$ & $C_2 \leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & 4x^2 \\ 1 & 3x & 9x^2 \end{vmatrix} + (x)(2x)(3x) \begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & 4x^2 \\ 1 & 3x & 9x^2 \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & x & x^2 \\ 1 & 2x & 4x^2 \\ 1 & 3x & 9x^2 \end{vmatrix} (1+6x^3)$$

$\Rightarrow (x-2x)(2x-3x)(3x-x)(1+6x^3) = (-x)(-x)(2x)(1+6x^3)$

$\Rightarrow \cancel{2}x^3(1+6x^3) = \cancel{10}\,5$

Let, $x^3 = t$

$\quad t(1+6t) = 5$

$\Rightarrow (6t-5)(t+1) = 0$

$\Rightarrow t = \frac{5}{6}, -1 = x^3 \Rightarrow x = \left(\frac{5}{6}\right)^{1/3}$ and (-1)

So only two distinct values of x exist

Q Which of following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha \;?$$

$$\Rightarrow \begin{vmatrix} 1+2\alpha+\alpha^2 & 1+4\alpha^2+4\alpha & 1+9\alpha^2+6\alpha \\ 4+\alpha^2+4\alpha & 4+4\alpha^2+8\alpha & 4+9\alpha^2+12\alpha \\ 9+6\alpha+\alpha^2 & 9+12\alpha+4\alpha^2 & 9+9\alpha^2+18\alpha \end{vmatrix} = -648\alpha$$

$$\Rightarrow \begin{vmatrix} 1 & \alpha & \alpha^2 \\ 4 & 2\alpha & \alpha^2 \\ 9 & 3\alpha & \alpha^2 \end{vmatrix} \times \begin{vmatrix} 1 & 2 & 1 \\ 1 & 4 & 4 \\ 1 & 6 & 9 \end{vmatrix} =$$

$$\Rightarrow \alpha \times \alpha^2 \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} \times 2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix}$$

LIMITS, CONTINUITY & DIFFERENTIABILITY

Limits

Q $\lim_{x\to\frac{\pi}{2}} \tan^2 x\left((2\sin^2 x+3\sin x+4)^{1/2}-(\sin^2 x+6\sin x+2)^{1/2}\right)$

Rationalization

$$\lim_{x\to\frac{\pi}{2}} \tan^2 x\,\frac{(2\sin^2 x+3\sin x+4-\sin^2 x-6\sin x-2)}{\underbrace{\sqrt{2\sin^2 x+3\sin x+4}}_{3}+\underbrace{\sqrt{\sin^2 x+6\sin x+2}}_{3}}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\tan^2 x(\sin^2 x-3\sin x+2)}{3+3} \Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\tan^2 x(\sin x-1)(\sin x-2)}{6}$$

$$\Rightarrow \lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x}{\cancel{\cos^2 x}}\,\frac{\overbrace{\cancel{(\sin x-1)(\sin x+1)}}^{(-1)}\overbrace{(\sin x-2)}^{(-1)}}{\underbrace{(\sin x+1)}_{2}\,6} \Rightarrow \frac{1}{12}$$

Q $\lim_{x\to\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right) = \lim_{x\to\infty}$

Substitution Put $x=\frac{1}{t}$, as $x\to\infty$, $t\to 0^+$

So, $\lim_{t\to 0^+}\left(\frac{1}{t^3}+\frac{3}{t^2}\right)^{1/3}-\left(\frac{1}{t^2}-\frac{2}{t}\right)^{1/2} \Rightarrow \lim_{x\to 0^+}\frac{(1+3t)^{1/3}-(1-2t)^{1/2}}{t}$

$\Rightarrow$ **L-H Rule**, $\Rightarrow \lim_{x\to 0^+}\frac{\frac{1}{3}(1+3t)^{\frac{1}{3}-1}(3)-\frac{1}{2}(1-2t)^{\frac{1}{2}-1}(-2)}{1}$

$\Rightarrow 2$

Q $\lim_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin x}-\sqrt[8]{1+\sin x}}\right)$ $\qquad (1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\ldots\infty$

Expansion

$$\lim_{x\to 0}\frac{x}{\left(\cancel{1}+\frac{1}{8}(-\sin x)+\frac{1}{8}\left(\frac{1}{8}-1\right)\frac{(\sin^2 x)}{2}+\ldots\right)-\left(\cancel{1}+\frac{1}{8}\sin x+\frac{1}{8}\left(\frac{1}{8}-1\right)\frac{\sin^2 x}{2}\ldots\right)}$$

$$\lim_{x\to 0}\frac{x}{-\frac{1}{4}\sin x+(\ )\sin^3 x+\ldots} = \lim_{x\to 0}\frac{x}{\sin x\left(-\frac{1}{4}+(\ )\sin^2 x+\ldots\right)}$$

$$\Rightarrow \lim_{x\to 0} \Rightarrow \frac{1}{-\frac{1}{4}+0} = -4$$

Q. $\lim_{n\to\infty} \frac{1}{1+n^2} + \frac{2}{2+n^2} + \dots + \frac{n}{n+n^2} + 2 \ (\neq 0)$

1) Sandwich Theorem

$\frac{1}{n^2+n} < \frac{1}{1+n^2} \le \frac{1}{1+n^2}$

$\frac{2}{n^2+n} < \frac{2}{2+n^2} < \frac{2}{1+n^2}$

$\vdots$

$\frac{n}{n^2+n} \le \frac{n}{n^2+n} < \frac{n}{1+n^2}$

Adding

$\frac{n(n+1)}{2n(n+1)} <$ Required $< \frac{n(n+1)}{2(n^2+1)}$

(Applying $\lim_{n\to\infty}$)

$\lim_{n\to\infty} \frac{1}{2} <$ Required $< \lim_{n\to\infty} \frac{n^2(1+\frac{1}{n})}{n^2 \cdot 2(1+\frac{1}{n^2})}$

$\downarrow \frac{1}{2}$ Hence Required $\to \frac{1}{2}$ $\quad \frac{1}{2}$

2) Limit of sum

$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r}{r+n^2} = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{r/n}{(\frac{r}{n})\frac{1}{n} + 1}$

Definite integral as limit of sum, $\Sigma \to \int$, $\frac{r}{n} \to x$, $\frac{1}{n} \to dx$

Continuity — Basics + Graph + Theorems

Q. $\lim_{x\to 0} x\left[\frac{4}{x}\right] = A$, then $f(x) = [x^2]\sin(\pi x)$ is discontinuous at,

a) $\sqrt{A+21} = 5$
b) $\sqrt{A} = 2$
c) $\sqrt{A+1} = \sqrt{5}$
d) $\sqrt{A+5} = 3$

$\lim_{x\to 0} x\left[\left(\frac{4}{x}\right) - \left\{\frac{4}{x}\right\}\right] \Rightarrow \lim_{x\to 0} 4 - \underbrace{x\left\{\frac{4}{x}\right\}}_{0 \times [0,1) = 0} = 4 = A$

checking limit $f(x)$ for $x = 5, 2, \sqrt{5}, 3$

at $x = \sqrt{5}$, $f(\sqrt{5}) = 5\sin(\sqrt{5}\pi)$

$f(\sqrt{5}^+) = 5\sin(\sqrt{5}\pi)$

$f(\sqrt{5}^-) = 4\sin(\sqrt{5}\pi)$ $\Big\} \neq$ Hence discontinuous at $x = \sqrt{5}$

Q. $f: R \to R$, $f(x) = \lim_{n\to\infty} \frac{\cos(2\pi x) - x^2\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ is continuous for $x \in$?

$$f(x) = \begin{cases} -\frac{\sin(x-1)}{x-1} & x \in (-\infty, -1) \\ -(1+\sin 2) & x = -1 \\ \cos(2\pi x) & x \in (-1, 1) \\ 1 & x = 1 \\ -\frac{\sin(x-1)}{x-1} & x \in (1, \infty) \end{cases}$$

at $x = -1$, RHL $= 1$, LHL $= -\frac{\sin 2}{2}$
R $\neq$ L

at $x = 1$, RHL $= -1$, LHL $= 1$
R $\neq$ L

Hence $f(x)$ is continuous $\forall\ x \in R - \{1, -1\}$

$\Rightarrow f(x) = [x^2] - 3$ $\quad [-\frac{1}{2}, 2] \to R$

$g(x) = [x^2-3](|x| + |4x+7|)$ $\quad [-\frac{1}{2}, 2] \to R$

suspicious points → $x = 0, 1, \sqrt{2}, \sqrt{3}, -\frac{1}{2}, 2$

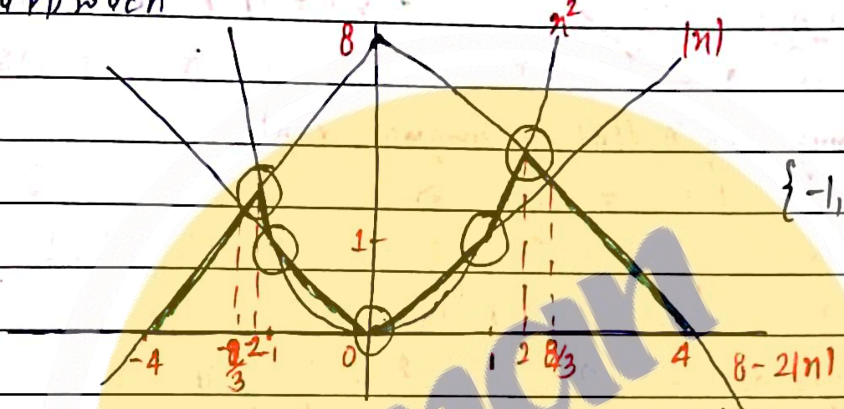
suspicious points, $x = 0, 1, \sqrt{2}, \sqrt{3}, -\frac{1}{2}, 2, 0 : -\frac{7}{4}$

(calculate LHD & RHD for Each Point to check differentiability) NOW

not each, use common sense, repeated roots, discontinuity → non diff

Q Let $f(x) = \begin{cases} \max(|x|, x^2), & |x| \le 2 \\ 8 - 2|x|, & 2 < |x| \le 4 \end{cases}$ in the interval $(-4, 4)$ f is not differentiable at ?

$\Rightarrow$ Graphical approach



$\{-1, 1, 0, 2, -2\}$

Q Let f be differentiable f^n from $R \to R$ such that $|f(x) - f(y)| \le 2|x-y|^{3/2}$ $\forall\, x, y \in R$, If $f(0) = 1$, then $\int f^2(x)\,dx = ?$

$\Rightarrow$

$|f(x) - f(y)| \le 2|x-y|\,|x-y|^{1/2}$

$\Rightarrow \left|\frac{f(x) - f(y)}{x-y}\right| \le 2|x-y|^{1/2}$

Both side $\lim x \to y$

$\lim_{x\to y}\left|\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y} 2|x-y|^{1/2}$

$|f'(y)| \le 0$

$\because |x| \le 0 \to x = 0$

$\therefore f'(y) = 0$ hence 'f' is constant

Given than $f(0) = 1$

So, $f(x) = 1$, Now $\int^1 f^2(x) = 1$

Let $c, k \in R$, if $f(x) = (c+1)x^2 + (1-c^2)x + 2k$ and $f(x+y) = f(x) + f(y) - xy$ $\forall\, x, y \in R$ then value of $|2(f(1) + f(2) + f(3) + --- + f(20))| = ?$

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Q. The slope of normal at any point (x,y), $x>0, y>0$ on curve $y=y(x)$ is given by $x^2/(xy - x^2y^2 - 1)$. If curve passes through $(1,1)$ then $e \cdot y(e) = ?$

$\Rightarrow$ $$-\frac{dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1} \Rightarrow -\frac{dy}{dx} = \frac{y}{x} - y^2 - \frac{1}{x^2}$$

$$\Rightarrow \frac{dy}{dx} - y^2 = \frac{1}{x^2} - \frac{y}{x} \longrightarrow -xy\,dx + x^2y^2\,dx + dx = x^2 dy$$

$$(x^2y^2+1)\,dx = x(x\,dy + y\,dx)$$

$$((xy)^2+1)\,dx = x\,d(xy)$$

$V = xy$

$$(v^2+1)\,dx = x\,dv$$

$$\int \frac{dv}{v^2+1} = \int \frac{dx}{x} \longrightarrow \tan^{-1} v = \ln|x| + C$$

$(1,1)$ $$\tan^{-1}(xy) = \ln|x| + C$$
$$C = \pi/4$$

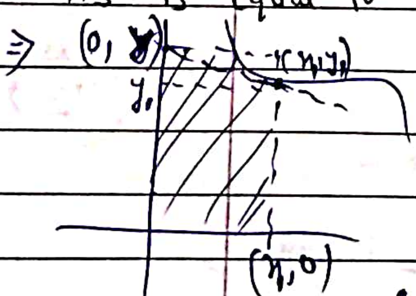
$x = e$ $$\tan^{-1} xy = \ln|x| + \pi/4$$

$$ey = \frac{\tan(\ln|e|) + \tan \pi/4}{1 - \tan \pi/4 \tan(\ln|e|)}$$

$$ey = \frac{\tan 1 + 1}{1 - \tan 1}$$

Q. Find eqn of curve which is such that area of rectangle constructed on the abscissa of any point and the intercept of tangent at this point on y axis is equal to 4.

$\Rightarrow$



say point be (x_1, y_1)

eqn of tangent, $$y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$y = \frac{dy}{dx}(x - x_1) + y_1 = \frac{dy}{dx}(-x_1) + y_1$$

area of rectangle $= |x_1 \times y| = 4$

$$= \left| x_1 \times \left((-x_1)\frac{dy}{dx} + y \right) \right| = 4$$

$$= \frac{dy}{dx} - \frac{y}{x} = \mp \frac{4}{x^2}$$

$$\text{I.F.} = e^{\int -\frac{1}{x} dx} = \frac{1}{|x|} = \frac{1}{x}$$

Q. If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation, $(x+1)\,dy = ((x+1)^2 + y - 3)\,dx$, $y(2) = 0$, then $y(3) = ?$

⇒ $(x+1)\,dy = ((x+1)^2 + (y-3))\,dx$

$(x+1)\,dy - (y-3)\,dx = (x+1)^2\,dx$

$$\int \frac{(x+1)\,dy - (y-3)\,dx}{(x+1)^2} = \int dx$$

$$\int d\left(\frac{y-3}{x+1}\right) = \int dx$$

$$\frac{y-3}{x+1} = x + c$$

$y(2) = 0$, $c = -3$

$y(3) = 3$

Q. Let $x = x(y)$ be sol^n of $2ye^{x/y^2}\,dx + (y^2 - 4xe^{x/y^2})\,dy = 0$ such that, $x(1) = 0$, $x(e) = ?$

⇒ $2y\,e^{x/y^2} + (y^2 - 4xe^{x/y^2})\,dy = 0$

$\cancel{e^{x/y^2}(2y - 4x)}$ $2e^{x/y^2}(y - 2x\,dy) + y^2\,dy = 0$

$$2e^{x/y^2}\frac{(y^2 - 2xy\,dy)}{y} + y^2\,dy = 0$$

$$2e^{x/y^2}\frac{(y^2 - 2xy\,dy)}{y^3} + \frac{y^2}{y^3}\,dy = 0$$

$\int$ both side $2e^{x/y^2}\,d(x/y^2) + \frac{dy}{y} = 0$

$2e^{x/y^2} + \ln|y| = c$

$x(1) = 0$, $c = 2$

$y = e$, $x = -e^2 \ln 2$

Q. If sol^n of diff eqn : $\frac{dy}{dx} + e^x(x^2-2)y = (x^2-2x)(x^2-2)e^{2x}$ satisfy $y(0) = 0$, Then $y(2) = ?$

a) $\frac{dy}{dx} + (e^x(x^2-2))y = (x^2-2x)(x^2-2)e^{2x}$ (Linear)

$$I.F. = e^{\int e^x(x^2-2)\,dx} = e^{\int e^x(\underbrace{x^2-2x}_{f} + \underbrace{2x-2}_{f'})\,dx} = e^{e^x(x^2-2x)}$$